When to switch from pasture to compost barn? An economic analysis of four scenarios in Minas Gerais, Brazil

Quando migrar do Sistema a pasto para o compost barn? Uma análise econômica de quatro cenários em Minas Gerais, Brasil

¿Cuándo cambiar de pasto a granero de compost? Un análisis económico de cuatro escenarios en Minas Gerais, Brasil

ABSTRACT
In Brazil, producers are migrating from pasture production systems to compost barns in search of greater profitability. These decisions are taken without an analysis of the cost,
economic feasibility, and risks involved in this change. Our study investigates the economic feasibility and risk of implementing the compost barn system and compares it to pasture production in the state of Minas Gerais’ southern region. Four scenarios were developed using data from literature and a panel of experts. They inform the production margin, the evaluated production system, investment data, revenue data and cost data. The scenarios were evaluated using a Monte Carlo simulation model. All scenarios had negative minimum values for NPV and IRR, indicating the probability of not being economically viable and demonstrating the risks involved in the dairy activity. However, scenarios 2 (Pasture, from 501 to 1,000 liters/day) and 4 (Compost Barn, from 1,000 to 2,000 liters/day) had positive means for NPV, with the highest probability of occurring values greater than zero. The analysis of the IRR result reinforces the better performance presented by scenarios 2 and 4, with greater probabilities of positive return and, consequently, lower risk. Scenarios 1 (Pasture, up to 500 liters/day) and 3 (Compost Barn, from 501 to 1,000 liters/day) can be discarded by the producer as an investment alternative.

Keywords: pasture system, compost barn, economic feasibility, risk analysis, Monte Carlo simulation model.

RESUMO
No Brasil, os produtores estão migrando dos sistemas de produção de pastagens para compostar celeiros em busca de maior rentabilidade. Estas decisões são tomadas sem uma análise do custo, viabilidade econômica e riscos envolvidos nesta mudança. Nosso estudo investiga a viabilidade econômica e o risco de implantação do sistema de celeiro de compostagem e o compara à produção de pastagens na região sul do estado de Minas Gerais. Quatro cenários foram desenvolvidos utilizando dados da literatura e um painel de especialistas. Eles informam a margem de produção, o sistema de produção avaliado, dados de investimento, dados de receita e dados de custo. Os cenários foram avaliados utilizando um modelo de simulação de Monte Carlo. Todos os cenários tinham valores mínimos negativos para o VAL e a TIR, indicando a probabilidade de não serem economicamente viáveis e demonstrando os riscos envolvidos na atividade leiteira. No entanto, os cenários 2 (Pastagem, de 501 a 1.000 litros/dia) e 4 (Celeiro de Compostagem, de 1.000 a 2.000 litros/dia) tiveram médias positivas para o VAL, com maior probabilidade de ocorrência de valores maiores que zero. A análise do resultado da TIR reforça o melhor desempenho apresentado pelos cenários 2 e 4, com maiores probabilidades de retorno positivo e, consequentemente, menor risco. Os cenários 1 (Pastagem, até 500 litros/dia) e 3 (Celeiro de Compostagem, de 501 a 1.000 litros/dia) podem ser descartados pelo produtor como alternativa de investimento.

Palavras-chave: sistema de pastagem, celeiro de compostagem, viabilidade econômica, análise de risco, modelo de simulação de Monte Carlo.

RESUMEN
En Brasil, los productores están migrando de sistemas de producción de pastos a graneros de compostaje en busca de mayor rentabilidad. Estas decisiones se toman sin un análisis del costo, la viabilidad económica y los riesgos que implica este cambio. Nuestro estudio...
investiga la viabilidad económica y el riesgo de implementar el sistema de graneros de compostaje y lo compara con la producción de pastos en la región sur del estado de Minas Gerais. Se desarrollaron cuatro escenarios utilizando datos de la literatura y un panel de expertos. Informan el margen de producción, el sistema de producción evaluado, datos de inversión, datos de ingresos y datos de costos. Los escenarios se evaluaron utilizando un modelo de simulación de Monte Carlo. Todos los escenarios tuvieron valores mínimos negativos para el VPN y la TIR, lo que indica la probabilidad de no ser económicamente viable y demuestra los riesgos que implica la producción lechera. Sin embargo, los escenarios 2 (Pastoreo, de 501 a 1.000 litros/día) y 4 (Granero de Compostaje, de 1.000 a 2.000 litros/día) tuvieron promedios positivos para el VAL, con mayor probabilidad de ocurrencia de valores mayores a cero. El análisis del resultado de la TIR refuerza el mejor desempeño que presentan los escenarios 2 y 4, con mayores probabilidades de retornos positivos y, en consecuencia, menor riesgo. Los escenarios 1 (Pastoreo, hasta 500 litros/día) y 3 (Granero de Compostaje, de 501 a 1.000 litros/día) pueden ser descartados por el productor como alternativa de inversión.

Palabras clave: sistema de pastos, granero de compost, viabilidad económica, análisis de riesgos, modelo de simulación Monte Carlo.

1 INTRODUCTION

Brazil is the fourth largest milk producer in the world, preceded by India, Pakistan, and the United States, with an estimated production of 36,752 tons in 2020 (FAO, 2021). The country has over 1 million milk-producing properties, with a predominance of small and medium farms, with approximately 4 million people. This production is concentrated mainly in the southern and southeastern regions of the country, with the state of Minas Gerais being the main producer contributing to approximately 27% of national production (MAPA, 2021).

There have been considerable changes in Brazilian dairy from the 1990s onwards. Such changes were caused by market deregulation, Mercosur's formalization, macroeconomic stability, new production and marketing structure, and the growing power and awareness of the consumer market—demanding increasing quality, prices, and variety (Bánkuti and Caldas, 2018; Vilela et al., 2017). Therefore, the dairy industry made structural and strategic changes, seeking to become competitive and innovative in the global market—pressuring milk producers to produce with higher quality and lower costs (Almeida and Bacha, 2021; Jung and Matte-Júnior, 2017). Milk producers are the weakest
part of the sector, since in the milk agribusiness there are typical situations of imperfect competition such as oligopoly and oligopsony, causing the producer to suffer more from the demands of the market (Fassio et al., 2005).

In this context, owing to the increased productivity of the herd, in the last two decades, milk production in Brazil has increased by almost 80%, using the same number of cows milked. Furthermore, there was a significant reduction in the number of producers and the intensification of production systems. Because of the adoption of new technologies, it is possible to significantly increase the productivity of animals, land, and labor, and consequently, the scale of production on farms (Rocha et al., 2020).

Increased productivity and farm management are related to the milk production systems. The typical system in Brazil is extensive in pastures, but there are more technified systems that use supplementation with forage and concentrates (semi-intensive), plus intensive confinement, especially in higher production and productivity regions (Sorio, 2018).

Among the intensive systems, the compost barn has been widely adopted in Brazil since it arrived in the country in 2011. This system comprises a large covered physical space for the cows to rest, with the primary objective of ensuring the animals' comfort and a dry place to stay during the year and the composting of bedding material (Mota et al., 2017). According to Leso et al. (2020) and Black et al. (2013), this system has a lower cost of implementation and maintenance, compared to other confined systems, and provides results equal to or superior to the others, combined with factors of better comfort and animal welfare.

The increase in milk production and the improvement in the herd’s health were the main factors that led producers in Minas Gerais to implement compost barns on their properties, according to research carried out by Oliveira et al. (2019). As stated by these authors, when compared to pasture systems, compost barns have higher implementation costs, higher operating costs, and, consequently, higher costs of milk production. However, it has a greater production per area (Silva et al., 2019). Therefore, efficient management by the producer is necessary, so that the gains from the greater production offset the higher costs with its implementation and operation.
Conversely, Zulpo and Carvalho (2020) and Kruger et al. (2019) compared the economic viability of compost barn with pasture systems and concluded that the former, despite being viable, has a lower return and a longer period, under the conditions studied, demonstrating that the simple adoption of a system with higher production is not necessarily the best alternative for producers. Furthermore, in Brazil, the compost barn systems showed great variability in the projects, dimensions, and materials used to implement the system, with different management forms—generating different costs and without effective support from agricultural research. Some of these characteristics are not following the values recommended in the literature, which can cause serious difficulties in handling and operating the system (EMBRAPA, 2021; Oliveira et al., 2019).

Therefore, in this context, analyzing the costs, economic feasibility, and risks of implementing a compost barn system are important to validate the investments that can be made by producers and to help in decision-making in dairy management. Furthermore, the difficulty of the producer in determining the right time and the best level of production to migrate from the pasture production system to the compost barn production system, which is economically viable, is the identified problem and the main motivation of this proposal.

Thus, the study investigates the economic feasibility and risk of implementing the compost barn system and compares it to pasture production in the state of Minas Gerais’ southern region. Using a Monte Carlo simulation model, four scenarios were developed and evaluated, involving the two production systems with different productivity levels.

2 MATERIALS AND METHODS

To achieve the proposed objectives, a literature review was initially carried out seeking information on technical indicators and descriptions of the production systems that would be evaluated. This review focused on the studied region—the south of the state of Minas Gerais. Next, a panel of experts was convened to elaborate on the scenarios based on the information found in the review. The panel comprised three technicians from “Minas Gerais Technical Assistance and Rural Extension Company” (EMATER-MG), who worked in the studied region and had experience in dairy cattle farming. Three
meetings were held during February 2021, when the experts specified the simulated properties in the four scenarios. Once the experts involved are knowledgeable about the topic, their opinions can be reliable.

Using the defined scenarios, a Monte Carlo simulation model was built. This technique allows us to account for risk in quantitative analysis and decision-making by building models of possible results. The model is built by substituting a range of values from a probability distribution for any factor that has inherent uncertainty. The results were then calculated continuously, each time using a different set of random values from the probability functions. Finally, this simulation process produces distributions of outcome values (Richardson, 2010; Mun, 2006).

Several authors have used Monte Carlo simulations for economic analyses in the dairy sector. Focker et al. (2021) estimated the direct short-term financial losses related to the 2013 aflatoxin incident for maize traders, the feed industry, and the dairy sector in the Netherlands. Krivko et al. (2021) studied the potential impacts of frequent price discounts, namely the loss of value-added tax (VAT) for the state budget due to low milk retail prices. Demeu et al. (2020) and Lopes et al. (2020) conducted economic feasibility studies for implementing new equipment and infrastructure. Finally, Shalloo et al. (2004) determined the influence of variation in milk price, concentrate cost, and silage quality on-farm profitability by modeling two calving patterns.

Following the approach proposed by Richardson (2010), the simulation model construction starts “from the top down,” with the determination of the output variables—those that the decision-maker considers important for solving the problem in question. Next, we determine the equations and parts of the model required to correctly calculate these output variables.

As in the present work, the objective is to assess the economic feasibility and the risk involved in implementing a pasture system and a compost barn system, where the output variables are the net present value (NPV) and the internal rate of return (IRR). NPV can be defined by Equation 1:

$$NPV = -C_0 - \sum_{n=1}^{N} \frac{CF_n}{(1+MARR)^n} (1)$$
Where:

\[ C_0 = \text{initial investment in period 0.} \]
\[ FC_n = \text{cash flow in period } n. \]
\[ MARR = \text{minimum attractiveness rate of return} \]
\[ n = \text{period, in which } n = 1, 2, ..., N. \]

The IRR can be defined as the rate that makes the NPV of a cash flow equal to zero. It is the discount rate that equals the sum of cash flows to the investment value. The IRR will then be the rate “\( i \)” that resets Equation 2:

\[
NPV = -C_0 - \sum_{n=1}^{N} \frac{CF_n}{(1+MARR)^n} = Zero \tag{2}
\]

Then, the intermediate variables that are necessary for calculating the output variables must be determined. Thus, from equations (1) and (2), we see that the MARR and the analysis period (\( n \)) will be variables determined by the user (exogenous variables to the model); therefore, they are not calculated by the same (endogenous variables). They can also be called “input variables” and their determination will be made following the study objective, with the experience and knowledge of those involved in the work, or the literature. In this case, the MARR was 3% and the analysis period was ten years. The initial investment (\( C_0 \)) is given by Equation 3:

\[
C_0 = \Sigma \text{infrastructure costs} + \Sigma \text{operational costs} \tag{3}
\]

These costs are determined by following the needs of rural property and market value. The cash flow (\( CF_n \)) is composed of Equation 4:

\[
FC_n = \text{total revenue}_n - \text{total operacional cost}_n \tag{4}
\]

The total cost is given by the sum of all costs involved in the activity and can be represented by (Equation 5):
Total operational cost = total fixed costs + total variable costs (5)

Total fixed costs were calculated by adding up the fixed costs for each year, which were defined by a panel of experts. Variable costs were divided into food, reproduction, and medication expenses. Feeding costs (FC) for the animal at life stages $t$ and $n$ were calculated using Equation 6:

$$FC_n = \sum^T_i \text{number of animals}_{tn} \times \text{daily consumption of the input}_{tn} \times \text{annual consumption time}_{tn} \times \text{input price}_n$$ (6)

Reproduction costs (RC) in period $n$ were calculated according to Equation 7:

$$RC_n = \text{number of cows}_n \times \text{amount of semen}_n \times \text{semen price}_n$$ (7)

Medication costs (MC) were defined by (Equation 8):

$$MC_n = \sum^T_i \text{number of animals}_{tn} \times \text{annual drug consumption}_{tn} \times \text{drug price}_n$$ (8)

All variables have their values defined, except for variables defined as stochastic feed ratio price and the number of animals. As criteria for this definition, a stochastic variables’ definition is used: (1) variables about which there is still uncertainty, even after the best prediction that could be conceived, and (2) those variables that decision-makers cannot control or predict.

The feed price was simulated considering a triangular probability distribution, as there was no historical data because of the diversity of brands and models. The average value defined was R$2.10\text{ }^1$, with the lowest expected value being R$1.89$, and the maximum value was R$2.31$. These values were defined in the expert panel. The number of

\[1\] All monetary values in this paper are in Brazilian Reais.
animals is given by the evolution of the herd, and the equations used for this calculation are shown in the Appendix.

For dairy farming, the total revenue (TR) involved the revenue from the sale of milk and animals, according to Equation 9, repeating it for each stage of life of the animals (calf, female, and male from 1 to 2 years, etc.):

\[ TR = (\text{quantity of milk produced} \times \text{milk price}) + (\text{number of animals sold} \times \text{average sale price of animals}) \] (9)

The quantity of milk produced (QMP) in period \( n \) was obtained according to Equation 10:

\[ QMP_n = \text{number of lactating animals}_n \times \text{average milk production}_n \] (10)

The average milk production (AMP) was obtained according to Equation 11:

\[ AMP_n = \text{lactation period of the animal} \times \text{average daily milk production} \] (11)

The number of animals sold (QAS) was obtained using Equation 12, repeating it for the animal's life stages:

\[ QAS_n = \sum_{t} \text{proportion of animal sales}_tn \times \text{total number of animals}_tn \] (12)

The number of lactating animals, the number of animals sold, and the sale price of the animals were defined in the expert panel. The price of milk and average daily milk production were considered as the stochastic variables. The price of milk for the different years was simulated considering the normal probability distribution with a standard deviation of 0.152604, centered on an average of R$2.00 for Scenario 1 and R$ 2.11 for the others, considering the quality of milk. This probability was defined after performing a Kolmogorov-Smirnov test from the annual average milk price data from 2005 to 2020.
(CILeite, 2021), obtaining p-value confidence of approximately $4.09 \times 10^{-17}$, within the 95% confidence standard commonly used.

The average daily milk production for each year was calculated considering the triangular distribution with mean and extremes defined according to the interview conducted with experts for each scenario: an average of 15 L, with less production of 10 L and greater production of 20 L for pasture systems; an average of 30 L, with less production of 25 L and greater production of 35 L for compost barn systems. A triangular distribution was defined because of the lack of historical data to calculate the distribution of the volume of milk produced.

After specifying the equations and defining the variables that will be treated as exogenous (input) and endogenous (variables calculated in the model), the model was assembled in an Excel spreadsheet. Subsequently, the simulation was carried out using the Python programming language, by means of the Xlwings library (XLWINGS, 2021) in Excel, and the Random library (PYTHON, 2021) to generate random numbers within the distribution. The analyses were performed also with this same language, using graphic libraries, Matplotlib (MATPLOTLIB, 2021) and Seaborn (SEABORN, 2021), along with the Pandas (The Pandas development team, 2020) and Numpy (NUMPY, 2021), libraries meant to assemble the tables. The Kolmogorov-Smirnov test was performed using the SciPy library (SciPy, 2021), and the standard deviation was calculated using the Pandas Library (The Pandas Development Team, 2020) for the Python programming language.

3 RESULTS AND DISCUSSION

The scenarios defined in the expert panel are presented in Table 1. They inform the production margin, the evaluated production system, investment data (land, semi-movements, improvements, and equipment), revenue data (milk sales price, animal sales price, animal production and sales indexes), and cost data (input prices, fixed cost values, depreciation values, and feeding, medical, and insemination indexes for formulas).
Table 1. Main characteristics of the scenarios according to the expert panel.

<table>
<thead>
<tr>
<th>Cost components</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production system</td>
<td>Pasture</td>
<td>Pasture</td>
<td>Compost Barn</td>
<td>Compost Barn</td>
</tr>
<tr>
<td>Daily productivity (liters)</td>
<td>Up to 500</td>
<td>From 501 to 1,000</td>
<td>From 501 to 1,000</td>
<td>From 1,000 to 2,000</td>
</tr>
<tr>
<td>Daily productivity (liters/cow)</td>
<td>10 to 20</td>
<td>10 to 20</td>
<td>25 to 35</td>
<td>25 to 35</td>
</tr>
<tr>
<td>Property size (hectares)</td>
<td>20</td>
<td>40</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>Implementation cost (R$)</td>
<td>1,038,296.00</td>
<td>2,238,492.00</td>
<td>2,734,468.00</td>
<td>4,813,636</td>
</tr>
<tr>
<td>Milk sales price (R$/liter)</td>
<td>2.00</td>
<td>2.11</td>
<td>2.11</td>
<td>2.11</td>
</tr>
<tr>
<td>Proportion of lactating cows (%)</td>
<td>75</td>
<td>75</td>
<td>83</td>
<td>83</td>
</tr>
<tr>
<td>Lactation period (days)</td>
<td>270</td>
<td>270</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Labor</td>
<td>Family and one hired (part time)</td>
<td>Family and one hired</td>
<td>Family and one hired</td>
<td>Family and six hired</td>
</tr>
<tr>
<td>Family maintenance (R$/year)</td>
<td>18,000.00</td>
<td>18,000.00</td>
<td>18,000.00</td>
<td>24,000.00</td>
</tr>
</tbody>
</table>

Source: Research data.

The investment required for implementing a compost barn system was greater in all cases, despite the smaller need for the area. This happens because of greater investments in improvements and equipment that are necessary for the system to function. Productivity in compost barn systems is also higher, as the breeds of semimovants expected for this type of enterprise have higher productivity. Family maintenance in Scenario 4 is greater, and a greater amount of family labor is necessary for this scenario. Table 2 summarizes the main statistics for the NPV and IRR results for the projected ten years using the Monte Carlo model. Ten thousand simulations were performed, as proposed by Richardson (2010), for the stochastic variables: milk price, average daily production volume per cow, number of animals, and feed price.

All scenarios had negative minimum values for NPV and IRR (Table 2), indicating the probability of not being economically viable and demonstrating the risks involved in the dairy activity. However, scenarios 2 and 4 had positive means for NPV, with the highest probability of occurring values greater than zero. Thus, Scenario 2 presents a 69.71% probability of being economically viable, whereas Scenario 4 presents 72.87%. Scenarios 1 and 3 have a small probability of being economically viable (6.43% and 0.8%, respectively).
Table 2. Summary statistics of NPV and IRR

<table>
<thead>
<tr>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Net Present Value – NPV (mil / USD)</strong></td>
<td><strong>NPV &gt; 0 (%)</strong></td>
<td><strong>Internal Rate of Return - IRR (%)</strong></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>- R$ 160.114,39</td>
<td>R$ 137.553,40</td>
<td>- R$ 400.420,60</td>
</tr>
<tr>
<td>StDev</td>
<td>R$ 105.436,82</td>
<td>R$ 266.678,60</td>
<td>R$ 165.755,60</td>
</tr>
<tr>
<td>Min</td>
<td>- R$ 542.201,42</td>
<td>- R$ 775.943,60</td>
<td>- R$ 1.092.569,00</td>
</tr>
<tr>
<td>Max</td>
<td>R$ 227.586,13</td>
<td>R$ 1.213.646,00</td>
<td>R$ 309.535,40</td>
</tr>
<tr>
<td>NPV &gt; 0</td>
<td>6,43%</td>
<td>69,17%</td>
<td>0,8%</td>
</tr>
</tbody>
</table>

Source: Research data.

The MARR used was 3%. Thus, the analysis of the IRR result reinforces the better performance presented by scenarios 2 and 4, with greater probabilities of positive return and, consequently, lower risk. Conversely, the IRR results show that pasture production system scenarios are more sensitive to variations in stochastic variables.

Figure 1 shows the cumulative probability distribution for NPV (in A) and IRR (in B) for the four simulated scenarios. In A, the probability of the project becoming unfeasible (NPV<0) corresponds to the value on the vertical axis associated with the intercept of this axis and the cumulative distribution curve. For example, in the second and third scenarios, the interception occurs near a probability of 0.3% or 30%, probability 0.3 has an IRR of -R$6.901.13. Comparing the four scenarios in Figure 1, it is possible to see that scenarios 1 and 3 obtain worse results than scenarios 2 and 4, the first ones with a higher probability of negative NPV values. Scenarios 2 and 3 have very close production and investment. However, Scenario 2 can be compared with Scenario 4 and Scenario 3 with Scenario 1 in terms of outcome. The producer should not produce in Scenario 3, considering the option of scenario 2 equivalent to better results economically.
Looking at Figure 1 B, where there is a cumulative result of probabilities according to IRR, one can see that stage 2 reaches a higher return for the producer than stage 4. However, stage 4 is maintained with more constant return results, whereas Scenario 2 has greater variations. Furthermore, considering scenarios 1 and 3, there are the same characteristics regarding its variation, with Scenario 1 varying more than Scenario 3. That is, scenarios 1 and 2 of the pasture production system are more sensitive to variations in the stochastic variables, as shown in Table 2, through the standard deviation.

Scenarios 1 and 3 can be discarded by the producer as an investment alternative because of the high probability of the NPV being less than zero and for presenting a return below the expectations for the risk. Scenarios 2 and 4 are feasible for the producer, considering that they have a higher probability of positive NPV values. Scenario 4 presents an even lower risk to the producer when considering the IRR. Thus, scenarios 2 and 4 are better options, with the first having the advantage of achieving higher NPV and having less initial investment required, while the second has a lower possibility of unfeasibility.
and less sensitivity to variations. The simulation results showed the superiority of pasture production systems for smaller production, and the compost barn production system needs to reach much larger production to have comparable viability.

4 CONCLUSIONS

Milk production in southern Minas Gerais is recognized for its family and small-producer characteristics, and the results of our study reinforce the possibility of this model generating wealth and income for the producer. Despite the high probability of success for two scenarios, one in pasture and one in compost barn, there is a need to reduce and manage the risks in the sector. Dairy farming in Brazil will continue to experience a high degree of risk and uncertainty from production, macroeconomic variables, demand, costs of production, and market price. Changes in the production system, without a prior and detailed study of their results, are also a considerable source of risk for producers. However, there is a high probability that the sector will be profitable. Future research should be directed at alternative risk management strategies that the sector could utilize, considering the many sources of risk identified in this research.
REFERENCES


https://seaborn.pydata.org/.


APPENDIX - HERD EVOLUTION EQUATIONS.

The number of cows in period $t$ is given by Equation 1. In this equation, the lactation rate is defined by the user with the cow breed average or expected.

$$\text{Cows}_t = (\text{cows} + \text{dry cows}_{t-1} + \text{female start breeding}_{t-1} + \text{bought cows}_t - \text{sold cows}_t - \text{female start breeding sold}_t - \text{dead cows}_t - \text{female start breeding dead}_t) \times \text{lactation rate}$$ (1)

The number of dry cows in period $t$ was obtained using Equation 2. The females at the beginning of reproduction were 1-2 years old for the compost barn system scenarios and 2-3 years old for the pasture system scenarios.

$$\text{Dry cows}_t = (\text{cows}_t - 1 + \text{dry cows}_t - 1 + \text{female start breeding}_t - 1 + \text{bought cows}_t - \text{sold cows}_t - \text{female start breeding sold}_t - \text{dead cows}_t - \text{female start breeding dead}_t) - \text{cows}_t$$ (2)

From Equation 3, we obtain the number of males or females up to one year in period $t$:

$$\text{Calves 1 year}_t = \frac{\text{cows}_t}{2}$$ (3)

The number of males or females from one to two years old in period $t$ is (Equation 4):

$$\text{Animal 1 to 2 years old}_t = \text{calves 1 year old}_t - 1 + \text{animal 1 to 2 years old purchased}_t - \text{calves 1 year old sold}_t - \text{calves 1 year dead}_t$$ (4)

On the other hand, the number of males or females aged 2 to 3 years in period $t$ is determined by Equation 5:

$$\text{Animal 2 to 3 years}_t = \text{animal 1 to 2 years old}_t - 1 + \text{animal 2 to 3 years old purchased}_t - \text{animal 1 to 2 years old sold}_t - \text{animal 1 to 2 years dead}_t$$ (5)

The amount of any animal purchased and the lactation rate are decided by the user, as well as the number of animals in $t_0$ (number of animals purchased at the beginning of the investment). The number of animals sold was obtained using equations 6 to 10.

$$\text{Discarded cows}_t = \text{cows}_{t-1} \times \text{cow culling rate}$$ (6)

$$1 \text{ year calf sold}_t = 1 \text{ year calf}_{t-1} \times 1 \text{ year calf sales rate}$$ (7)

$$\text{Animal 1 to 2 years old sold}_t = \text{Animal 1 to 2 years old}_{t-1} \times \text{sales rate of Animal 1 to 2 years old}$$ (8)
Animal 2 to 3 years old sold, \( t \) = Animal 2 to 3 years old \( t-1 \) × sales rate of Animal 2 to 3 years old (9)

The number of dead animals is known by the equations 10 to 13:

\[ \text{Dead cow}_t = \text{Cows}_t \times \text{cow death rate} \] (10)

\[ \text{Calves 1 year dead}_t = \text{calves 1 year}_t \times \text{calves death rate 1 year} \] (11)

\[ \text{Animal 1 to 2 years dead}_t = \text{Animal 1 to 2 years old}_t \times \text{death rate of animal 1 to 2 years} \] (12)

\[ \text{Animal 2 to 3 years dead}_t = \text{Animal 2 to 3 years old}_t \times \text{death rate of animal 2 to 3 years} \] (13)

The death/sale rates and the number of bulls are defined by the user, as well as the number of animals in \( t(0) \) (number of animals in the investment).