A case study on neuro-fuzzy architectures applied to the system identification of a reduced scale fighter aircraft

Um estudo de caso sobre arquiteturas neuro-difusas aplicadas à identificação do sistema de uma aeronave de caça em escala reduzida

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ABSTRACT
This study investigates different architectures of Neuro-Fuzzy applied to unsteady aerodynamic modeling based on experimental data from a reduced-scale aircraft, known as Generic Future Fighter. The comparison is made considering different fuzzy inference methods, membership function shapes, number of membership functions to describe the input variables and different output functions, in the case of Takagi-Sugeno inference method. All these comparisons are made using the differential evolution as an optimization tool. In the end, the results present the best Neuro-Fuzzy configuration applied to the system identification of the GFF. Furthermore, the conclusion presents insights about the possible future implementation of the methodology.

Keywords: neuro-fuzzy, experimental flight data, machine learning, unsteady aerodynamic modeling.

RESUMO
Este estudo investiga diferentes arquiteturas de Neuro-Fuzzy aplicadas à modelagem aerodinâmica instável com base em dados experimentais de uma aeronave de escala reduzida, conhecida como Generic Future Fighter. A comparação é feita considerando diferentes métodos de inferência difusa, formas de função de associação, número de funções de associação para descrever as variáveis de entrada e diferentes funções de saída, no caso do método de inferência Takagi-Sugeno. Todas essas comparações são feitas
INTRODUCTION

Machine Learning (ML) tools have increasingly become a useful tool in the field of aerodynamic modeling and system identification. The approach offers sophisticated ways to handle complex, high-dimensional data and provide insights about aerodynamics behavior.

The convolutional neural networks (CNNs) are adept at handling large-scale structured data, such as images, and are particularly efficient due to their structure which reduces the number of parameters through convolution and pooling operations. This reduction in parameters is important in managing the high-dimensional input parameters often encountered in neural networks. The work presented by Zan, et al. (2022) use modified CNN architectures to identify the aerodynamic model of a hypersonic wing to demonstrate the CNN-based modeling method.

Another technique, the Adaptive Network-based Fuzzy Inference System (ANFIS), combines neural networks and fuzzy logic to create a modeling tool. This approach is useful for regression and predictive modeling, where it converts input variables into fuzzy sets and projects them based on fuzzy membership. The work published by Brandon and Morelli (2012) demonstrates the application of the Neuro-Fuzzy applied to aerodynamic modeling of a real-scale aircraft. Due to the efficiency of the method during the validation steps, the author later published the results of an on-board identification, performing the validation during the flight (BRANDON AND MORELLI, 2016).

Deep learning (DL) techniques are also used in aerodynamic modeling and optimization. For example, Li, Kou and Zhang (2022) used DL to model the aerodynamic
distribution of a transonic wing, using numerical and experimental results to train the ANN. The authors consider that high-fidelity data is obtained experimentally, and low-fidelity data is obtained through numerical simulations, and they found that using only a small portion of high-fidelity data mixed with a large portion of low-fidelity data was sufficient to train a deep neural network (DNN) and obtain a good aerodynamic model.

This work intends to explore different configurations of Neuro-Fuzzy architecture, by varying some parameters, like membership function shapes, inference methods, number of membership functions for input variables and the output functions in the Takagi-Sugeno inference method.

2 METODOLOGY

This section contains an explanation of the methodology used in this case study. First, the Fuzzy Logic theory is explained, followed by the Fuzzy Rule-based System (FRBS) with an explanation of the most used inference methods. Finally, the Neuro-Fuzzy and the Differential Evolution are presented.

2.1 FUZZY LOGIC

The fuzzy logic can be used in the modeling of vast biological behaviors, like the growth of violets in the function of water and sunlight, as presented by Jafelice, Barros and Bassanezi (2012).

The fuzzy logic differs from the classical Boolean logic mainly because the latter can assume only two conditions, 0 or 1, while the first can assume several values varying between 0 and 1.

A simple example to illustrate fuzzy logic is the classification of people into the category of elderly. Depending on the jurisdiction, the law often designates individuals over the age of sixty as elderly. Using Boolean logic, only individuals who meet this exact age criterion are considered elderly. Fuzzy logic, however, introduces a different perspective by assigning a degree of membership that increases with age, as shown in Figure 1.
2.2 FUZZY RULE-BASED SYSTEM (FRBS)

The FRBS has four important structures to present, which can be described on the following items:

- **Input Processor:** The input processor converts real numbers into numbers with a certain degree of membership on fuzzy sets. This process is known as fuzzification.

- **Rule bases:** The rule bases are linguistic interpretations of the system behavior and combined with the inference machine, could be considered the core of FRBS. They can be created according to specialist knowledge through prepositions of the type IF...THEN. This step is responsible for establishing the relations between the linguistic variables.

- **Inference Machine:** The inference machine is responsible to establish the correlation between the input fuzzy sets with the output fuzzy functions (Takagi-Sugeno method), and this association is guided by the rule bases knowledge.

- **Output Processor:** The output processor is responsible to perform the defuzzification, which is to convert the degree of membership of the fuzzy sets into real numbers again.

The FRBS architecture can be seen in Figure 2.

Source: Adapted from classes notation from Prof. João Alberto Fabro and André Schneider de Oliveira.
2.2.1 Mamdani Inference Method

According to Jafelice et al. (2004), the defuzzification in the Mamdani inference method is made through the sum of areas underneath the curve. The resultant area is analyzed and the answer of the system is the position of the center of gravity of the total area. The exemplification of Mamdani’s inference method is shown in Figure 3.

To describe the Mamdani inference method, it is necessary to propose two arbitrary rules:

Rule 1: If (X is \( A_1 \) AND Y is \( B_1 \)) THEN (Z is \( z_1 \))

Rule 2: If (X is \( A_2 \) AND Y is \( B_2 \)) THEN (Z is \( z_2 \))
2.2.2 Takagi-Sugeno Inference Method

To better describe the TS inference method, the two following rules are proposed Jafelice et al (2003). The architecture of TS inference method is shown in Figure 4.

Rule 1: If (X is $A_1$ AND Y is $B_1$) THEN (Z is $z_1$)

Rule 2: If (X is $A_2$ AND Y is $B_2$) THEN (Z is $z_2$)
The defuzzification is made by a weighted average between the output functions multiplied by the weights estimated by the rule bases.

2.2.3 Poundered Individual Analysis Inference Method

The Poundered Individual Analysis (PIA) was first presented by Pereira, Jafelice and Finzi (2022), and it is very similar to the Mamdani inference method. Nonetheless, it does not have the area integration anymore, which gives this technique the ability to save some computational cost and the possibility to maintain a high interpretability of the linguistic variables.

The technique is based on the mathematical translation of each fuzzy proposition composing the rule bases. According to Pereira, Jafelice and Finzi (2022) to perform the PIA method, it is necessary to understand the following nine steps.

1. The definition of the FRBS inputs and outputs fuzzy sets, and the rule bases, have the same reasoning adopted in the Mamdani inference method.
2. The point where the membership function has the maximum value (one) is called the center point (CP). In case that exists a membership function with more than one CP, it is necessary to make an arithmetic mean between these points to identify the resultant CP.
3. For each fuzzy rule from FRBS, the effect of increasing or decreasing the input variable (IN) will perform a different effect on the output variable (OUT).
according to what type of relationship they have, i.e., directly proportional or inversely proportional.

- **Direct:** The representation when the input variable is directly proportional to the output variable will be IN \( \text{Dir} \) OUT.
- **Inverse:** When the correlation between IN and OUT are inversely proportional, the representation will be IN \( \text{Inv} \) OUT.
- **Neutral:** To conclude, when it is not possible to infer the relationship between IN and OUT, the representation will be IN \( \text{Neut} \) OUT.

4. For each certain input variable \( x_i \), which has a degree of belonging equal to \( \mu \) in a determined membership function, the same degree of belonging \( \mu \) will be projected to the correspondent output variable \( y_i \) membership function according with the rule base established \( (R_i) \), see

5. Figure 5. The projection provides two possible output candidates \( out_{\text{cand}_1} \) and \( out_{\text{cand}_2} \).

![Figure 5: PIA inference system candidate evaluation.](image)

Source: Adapted from Pereira, Jafelice and Finzi (2022).

6. In terms of selecting the correct output candidate, it is necessary to consider the following conditions:

- If in \(< CP \) and IN \( \text{Dir} \) OUT then \( out = out_{\text{cand}_1} \).
- If in \( > CP \) and IN \( \text{Inv} \) OUT then \( out = out_{\text{cand}_1} \).
- If in \(< CP \) and IN \( \text{Inv} \) OUT then \( out = out_{\text{cand}_2} \).
- If in \( > CP \) and IN \( \text{Dir} \) OUT then \( out = out_{\text{cand}_2} \).
• Other cases are threaten as an arithmetic mean between \( \text{out}_{cand_1} \) and \( \text{out}_{cand_2} \).

7. The process is repeated for each input variable. In terms of the rule bases \( R_i \), it is possible to define and output contribution \( \text{cont}_{Cjm} \), according to Eq. 1, where \( C_j \) is the output fuzzy set, \( m \) is the number of times that \( C_j \) had been already correlated in the rule bases definition. Also, \( \text{out}_{ki} \) is the out value chosen and \( a_k \) is the weight associated with the output variable, both in terms of a \( k \) input variable. When all variables have the same effect on the output process, it is assumed that \( a_k = 1 \).

\[
\text{cont}_{Cjm} = \frac{\sum_{k=1}^{n_{in}} a_k \text{out}_{ki}}{\sum_{k=1}^{n_{in}} a_k} \quad (1)
\]

8. The next step after defining the contribution of each rule, is to calculate the value of the consequent function \( f_{C_j} \) of each output fuzzy set \( C_j \). The Eq. 2 shows the calculation for the consequent function, where \( T_j \) is the sum of the rules associated with the set \( C_j \).

\[
f_{C_j} = \frac{\sum_{m=1}^{T_j} w_{Cjm} \text{cont}_{Cjm}}{\sum_{m=1}^{T_j} w_{Cjm}} \quad (2)
\]

9. To identify the weights, it is necessary to understand that the FRBS interprets the union as the minimum and the intersection as the maximum. Therefore, if a linguistic rule processes a condition with AND, the weight will be the larger value (max), and if the rule processes a condition with OR, the weight will be the smaller one (min). The Eq. 3 gives an example of the weight identification from an arbitrary rule, where \( w_{Cjm} \) is the weight of each rule \( R_i \) and \( W_{Cjm} \) is the weight of each consequent function \( f_{Cj} \).
\[ W_{Cj} = \max \left( w_{Cjm} \right), m = 1, \ldots, T \]

10. Finally, the output of the FRBS can be obtained through defuzzification. For PIA, the defuzzification is done by the weighted mean method. Eq. 4 shows the calculation, where \( n_c \) is the number of fuzzy sets from \( Z \) output.

\[
z = \frac{\sum_{j=1}^{n_c} W_{Cj} f_{Cj}}{\sum_{j=1}^{n_c} W_{Cj}}
\]

2.3 NEURO-FUZZY

The Neuro-Fuzzy was first presented by Jang (1993), and it is a combination of the interpretability of the Fuzzy Logic and the adaptability of the Artificial Neural Networks.

In this section, the Neuro-Fuzzy architecture is presented, followed by the explanation of each layer separately. In Fig. 3 it is possible to see the layers that compose the Neuro-Fuzzy architecture.

Figure 3: Neuro-Fuzzy architecture.

Layer 5
Layer 4
Layer 3
Layer 2
Layer 1
Source: Adapted from Pereira et al. (2017).
**Layer one:** Each node in this layer receives a single input variable $I_i(k)$ for the training process. The output of the $i_{th}$ node in the first layer at time $k$, denoted as $u_i^{(1)}(k)$, is equal to the input variable $I_i(k)$.

$$u_i^{(1)}(k) = I_i(k) \quad (5)$$

**Layer two:** In this layer the fuzzification of input variables is performed, that is, the real numbers are transformed into Fuzzy subsets with a certain degree of pertinence. The membership functions (MF) are built for the description of the inputs. Considering that the membership functions are approximated by Gaussian functions, the output of node $ij$ from layer 2 at time $k$, $u_{ij}^{(2)}(k)$, is given by:

$$u_{ij}^{(2)}(k) = e^{-\frac{(u_i^{(1)}(k)-m_{ij}(k))^2}{2\sigma_{ij}(k)}} \quad (6)$$

**Layer three:** The rule bases consist of prepositions of the type IF…THEN…, forming the correlation between input sets and output functions of the Adaptive Neuro-Fuzzy Inference Systems (ANFIS). For each rule, the logical operator AND and OR are applied, which corresponds to the minimum and maximum respectively. The output from the L node of the third layer ($u_L^{(3)}(k)$) is a function of the layer 2 for the selected output from rule $R_L$.

**Layer four:** The nodes from this layer are known as consequent nodes, and they are defined as a function (output function) $f_L: R^n \rightarrow R$ in which $f_L = f(I_1, ..., I_l, ..., I_n, w_{1L}, ..., w_{jL}, ..., w_{oL}, k)$, where $w_{1L}, ..., w_{oL}$ are weights that are determined in the ANFIS training optimization. Thereby, the output from node $L$ of the fourth layer, $u_L^{(4)}(k)$, is calculated as:

$$u_L^{(4)}(k) = u_L^{(3)}(k) f_L(I_1, ..., I_l, ..., I_n, w_{1L}, ..., w_{jL}, ..., w_{oL}, k) \quad (7)$$
Layer five: The last layer is responsible for providing the ANFIS result, through a pondered weight estimation given by the following equation:

\[ O(k) = \frac{\sum_{L=1}^{R} u_L^{(4)}(k)}{\sum_{L=1}^{R} u_L^{(3)}(k)} \]  

(8)

2.4 DIFFERENTIAL EVOLUTION

Differential Evolution (DE) was first published by Storn and Price (1997) and since then it has been widely used by the scientific community. Because of that, this optimization methodology was chosen to optimize the Fuzzy Logic parameters. According to Storn and Price (1997), the mutation probability (F) needs to be inside the interval \([0, 2]\) and the crossing probability (CR) \(\in [0, 1]\), and the value selection has to be determined by the user. Therefore, the optimization parameters used to acquire the results can be seen in Table 1.

<table>
<thead>
<tr>
<th>Optimization Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of individuals</td>
<td>150</td>
</tr>
<tr>
<td>Number of variables</td>
<td>17-25</td>
</tr>
<tr>
<td>Crossing probability (CR)</td>
<td>0.95</td>
</tr>
<tr>
<td>Mutation probability (F)</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Source: Developed by the authors (2024).

Using the DE optimization method to train the Neuro-fuzzy parameters, it took less than a minute (30 seconds) to optimize seventeen variables using more than three hundred data. The Fig. 4 depicts the simplified architecture of the differential evolution applied in this work.
2.5 GENERIC FUTURE FIGHTER (GFF)

The Generic Future Fighter (GFF) is a 13% scale representation of a 5th generation fighter, and conceptual design was developed by SAAB aeronautics. The subscale aircraft was built by Linköping University (JOUANNET et al., 2012). For this aircraft, Sobron et al. (2016) created a low-cost instrumentation to acquire the aerodynamic data through experimental flights. The aircraft is equipped with onboard devices, e.g., Pixhawk, which records the acceleration, the attitude angles rate, the velocity, the sideslip angle, the angle of attack, and more. The Figure 6 shows the 6 degrees-of-freedom (DOF) of the GFF.

According to Fossen (2011), the forces that acts in the non-inertial body axes of the aircraft can be obtained with the Equations 5, 6 and 7.

\[
\begin{align*}
\alpha_{Xcg} &= \frac{X}{m} = (\dot{U} + QW - RV + g \sin \theta) \\
\alpha_{Ycg} &= \frac{Y}{m} = (\dot{V} + UR - WP - g \cos \theta \sin \phi) \\
\alpha_{Zcg} &= \frac{Z}{m} = (W + VP - QU - g \cos \theta \cos \phi)
\end{align*}
\]
Figure 6: Generic Future Fighter non-inertial body axes.

Source: Developed by the authors (2024).

3 RESULTS

This section presents the results from the different Neuro-Fuzzy architectures. The section is divided in four subsections, the comparison between membership functions shapes, the comparison between fuzzy inference systems, the comparison between the number of membership functions for each input variable and the comparison between different output consequent functions.

3.1 COMPARISON BETWEEN MEMBERSHIP FUNCTION SHAPES

This section presents a comparison between three shapes of membership functions using PIA as a fuzzy inference system. The MF shapes analyzed were triangular, trapezoidal and Gaussian functions. Also, the number of membership functions of each input and output variable is maintained between the analysis. This analysis uses experimental data from GFF flight tests performed by Rueda (2021).
Figure 7: Comparison between membership functions shapes. Triangular, trapezoidal and Gaussian, respectively.

The results presented in Figure 7 emphasize that the best membership function shape for this application is the Gaussian membership functions.

3.2 COMPARISON BETWEEN FUZZY INFERENCE METHODS

Each comparison is made using a MISO (Multi Input Single Output) system with emphasis on the vertical force coefficient $C_Z$ because it is the most expressive degree of freedom among all. The Neuro-Fuzzy configuration counts with two input variables, which are the angle of attack ($\alpha$) and the elevator deflection ($\delta e$).

3.2.1 Mamdani Inference Method

The first evaluation was performed using the Mamdani fuzzy inference method. The time to perform the optimization is extremely high due to the integration of the area. Nevertheless, applying the Mamdani fuzzy inference method to the Neuro-Fuzzy with
DE also guaranteed a good curve fitting for the training section. Figure 8 illustrates the training and the validation graph using the Mamdani fuzzy inference method.

![Figure 8: Training and validation using Mamdani inference method.](image1)

Source: Developed by the authors (2024).

Due to the high computational cost followed by a not good validation, the Mamdani inference method is worse than the Takagi-Sugeno inference method.

### 3.2.2 Takagi-Sugeno Inference Method

This fuzzy inference method allows to reduce the number of fuzzy parameters in the optimization function because the consequent function can be considered as a single variable, which can save some computational effort and time. The Figure 9 presents the results obtained through training and validation using the Takagi-Sugeno inference system.

![Figure 9: Training and validation using Takagi-Sugeno inference method.](image2)

Source: Developed by the authors (2024).
3.2.3 Pondered Individual Analysis Inference Method

Here, a new fuzzy inference method (PEREIRA; JAFELICE; FINZI, 2022) is applied to the Neuro-Fuzzy with Differential Evolution. The curve fitting for the training and validation are shown in Figure 10 and the coefficient of determination for the training of each fuzzy inference method is shown in Table 2.

Figure 10: Training and validation using PIA inference method.

![Figure 10: Training and validation using PIA inference method.](source)

Table 2: Comparison between the three fuzzy inference method.

<table>
<thead>
<tr>
<th>Method</th>
<th>Training time [s]</th>
<th>Validation time [s]</th>
<th>$R^2$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mamdani</td>
<td>$1.7210 \times 10^3$</td>
<td>0.558</td>
<td>97%</td>
</tr>
<tr>
<td>TS</td>
<td>$0.206 \times 10^3$</td>
<td>0.026</td>
<td>97%</td>
</tr>
<tr>
<td>PIA</td>
<td>$3.052 \times 10^3$</td>
<td>0.176</td>
<td>95%</td>
</tr>
</tbody>
</table>

Source: Developed by the authors (2024).

3.3 INPUT NUMBER OF MEMBERSHIP FUNCTIONS

This section examines the number of membership functions (MF) of the input variables that can affect the model in terms of training curve fitting and evaluates the variation of the coefficient of determination ($R^2$).

Three different amounts of MF were trained for the input variables of the yaw moment coefficient ($C_n$). Analyzing the Figure 11, there is no visual difference between the increase in the amount of input MF.
The Table 3 shows that the coefficient of determination does not change significantly to justify the use of more than three MF. According to Brandon and Morelli (2016), the number of membership functions of the input variables should vary according to the correlation between the input and the output variables proportionally.

<table>
<thead>
<tr>
<th>Output Consequent</th>
<th>$R^2$</th>
<th>Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two MF</td>
<td>3</td>
<td>0.569</td>
</tr>
<tr>
<td>Three MF</td>
<td>5</td>
<td>0.570</td>
</tr>
<tr>
<td>Four MF</td>
<td>7</td>
<td>0.567</td>
</tr>
</tbody>
</table>

Source: Developed by the authors (2024).

3.4 DEPENDENCE OF THE INPUT VARIABLES IN THE OUTPUT FUNCTION

According to Jafelice et al. (2003), the output functions from the Takagi-Sugeno inference method are functions that depend on the input variables plus a constant. The comparison presented in this section evaluates the output function with the input variables and compares it to the output function described as a constant only.
After this case study, the authors concluded that the best membership function to identify the aerodynamic model from the GFF is the configuration presented in Table 4: Optimal Neuro-Fuzzy Configuration.

Table 4: Optimal Neuro-Fuzzy Configuration.

<table>
<thead>
<tr>
<th>Configuration Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inference method</td>
<td>Takagi-Sugeno</td>
</tr>
<tr>
<td>Membership function shape</td>
<td>Gaussian</td>
</tr>
<tr>
<td>Number of input variables</td>
<td>3</td>
</tr>
<tr>
<td>Number of input MF</td>
<td>3</td>
</tr>
<tr>
<td>Number of output functions</td>
<td>7</td>
</tr>
<tr>
<td>Output function type</td>
<td>Constant</td>
</tr>
</tbody>
</table>

4 CONCLUSION

This work presented a several configurations using the Neuro-Fuzzy with Differential Evolution applied to system identification, specifically the identification of the unsteady aerodynamic model of a subscale fighter aircraft.

The results show that the best configuration for this purpose is the Neuro-Fuzzy with the characteristics presented in Table 4. However, this is not the best configuration for every case.

Due to the time efficiency of the model validation, it is possible to perform the on-board validation of the system identification. As a future work, the authors will try to
integrate the model to an acquisition board system embedded in UAS (Unmanned Aircraft System).

Also as future works, the authors aim to analyze the uncertainties of the model and analyze the possibility of self-adaptation with online validations.

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REFERENCES


Sobron, A., Lundström, D., Staack, I. and Krus, P., 2016. “Design and testing of a low-cost flight control and data acquisition system for unstable subscale aircraft.” In 30th Congress of The International Council of the Aeronautical Sciences (ICAS), Daejeon,
Korea, September 25-30, Daejeon, South Korea. The International Council of the Aeronautical Sciences.
