Analysis of the influence of fillet geometry on stress concentration in keyway by the finite element method

Análise da influência da geometria do filete na concentração de tensão no keyway pelo método dos elementos finitos

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ABSTRACT
Machines play a pivotal role in the modern world, as they streamline various activities by virtue of their capacity to convert energy, such as transforming electrical energy into
mechanical energy. In this context, the mechanical shaft assumes significant importance, as it enables the transmission of power to a wide range of applications. Being a solid component, the shaft is subjected to mechanical stresses, and the keyway groove, present in the shaft, is a feature that exacerbates stress concentrations in its geometry, which is undesirable as it increases the risk of failure due to rupture. Therefore, this research aimed to investigate the influence of the keyway groove geometry on stress concentration in shafts under bending loads, using simulations based on the finite element method. The study compares elliptical keyways with circular keyways and assesses how variations in parameters impact stress concentration, employing the Ansys tool for the analysis. The results demonstrate that slight modifications in the keyway groove geometry can significantly reduce stress concentrations, thereby contributing to the optimized design of mechanical components and providing valuable insights for engineers working in this field.

**Keyword:** elliptical fillet, stress amplification, numerical simulation, mechanical drawing.

1 INTRODUCTION

In the modern world, machines have been increasingly integrated into everyday activities. In the mechanical industrial sector, for instance, machines often operate
through the use of electric motors (ISTIAQUE, 2020). The power generated at the motor's drive shaft needs to be transmitted to other rotating components within the system. To accomplish this, specific parts designed to transmit torque and rotation are utilized.

One such component is the key, commonly used to transfer torque and rotation between elements. It is affixed to the shaft and the other component to prevent relative rotation and follows standardized design guidelines, with the parallel key being the most prevalent variation (DIN 6885-1, 1968). However, during coupling, both the shaft and the other rotating component must be machined, which results in stress concentration at the created slot.

Shafts are mechanical elements designed to transmit torque to attached equipment and can be subjected to axial, torsional, and flexural stresses. Keyway slots are stress concentrators in shafts, and high mechanical stresses are undesirable as they impact the expected performance in the design, potentially significantly increasing safety factors for static and dynamic loads (OZKAN and ERDEMIR, 2020). Pedersen (2010) investigated the reduction of stress concentration in parallel keyway slots subjected to torsional loads through modifications in the fillet geometry. It was found that the fillet geometry minimizing stress concentration by approximately 50% can be represented as an ellipse.

Given the aforementioned context, the present study aims to investigate the influence of elliptical fillets on stress concentration in parallel keyway slots subjected to bending loads through numerical simulations. This research seeks to establish optimized patterns that reduce stress gradients along the component's body.

2 FINITE ELEMENT METHOD

The Finite Element Method (FEM) is a numerical technique designed to approximate solutions to complex problems described by ordinary differential equations and partial differential equations. It achieves this by subdividing the geometry into multiple smaller elements, which are then organized into a matrix for calculating the approximation at specific points, as elaborated by Benito et al. (2023).

Through discretization, a continuous system is divided into a finite number of smaller elements, with the intersection point of these elements referred to as a node. The
mesh is the set of elements used in the discretization of the structural geometry and, therefore, it is crucial to define the type of geometry and the shape that the mesh will take, as discussed by Kwon and Bang (2018). FEM is commonly employed in structural calculations due to its inherent nature, where the displacement of nodes provides insights into the behavior of each element, as highlighted by Alves Filho (2009). In an FEM mesh, the result of one element does not influence another, meaning that if one region has more refinement while another has less, it will lead to better results for the former and less accurate results for the latter, in accordance with Zienkiewicz and Taylor (2005).

3 MATERIALS AND METHODS

3.1 SHAFT AND KEYWAY

To create the geometry under analysis, we used the SolidEdge software, which is available for free for academic use. For the axis being analyzed, a diameter of 28 mm was assigned. According to DIN 6885-1 (1968) standards, the keyway geometry was defined with $W = 8$ mm and $H = 7$ mm in relation to the diameter. We also determined that the length of the keyway would be equal to its width, so $W = L = 8$ mm.

Figure 1 – Shaft geometry for simulation. a) Full view, b) Section A-A

To establish the length of the shaft, we applied the Saint-Venant principle, which states that loads applied to a body cause purely local effect. As a result, stress rapidly...
decreases with increasing distance (NAKAMURA and LAKES, 1995). Based on this principle, stress redistributes after a discontinuity, tending toward an average stress value. This average depends on the distance to the discontinuity. Therefore, we conducted several simulations, varying the shaft length, to ensure that the bending stress generated on the right side of the shaft was equal to that on the left side, making them match the nominal bending stress. In other words, this accounts for the absence of stress concentration resulting from the keyway slot. Thus, the keyway geometry on the shaft is depicted in Figure 1.

3.2 ELLIPTICAL AND CIRCULAR FILLETS

In the case of the circular fillet, initially, the calculation of stress concentration will be performed numerically. This will allow for the comparison of these results with those found in the literature and the selection of boundary conditions and mesh parameters to ensure accurate results.

As for the elliptical fillet, its geometry parameters are defined based on the circular fillet, where a circular fillet with $r = 0.4$ mm was chosen as a comparison parameter for the elliptical fillets. Figure 2 presents the details of both types of fillets.

Figure 2 – Geometric parameters of the fillets: a) elliptical, b) circular

Source: Author (2023)
In the analysis of the elliptical fillet, parameter $b$ will be held fixed at 0.4 mm, while parameter $a$ will be varied to achieve different $a/b$ ratios within the range of 0.5 to 2. It is important to note that an $a/b$ ratio equal to 1 result in a circular fillet with $r = 0.4$ mm, allowing for a comparison between numerical values and literature data.

3.3 NUMERICAL SIMULATION

For the numerical simulation, we used the Ansys software, which offers a free student version. Through this software, we generated the mesh and defined the boundary conditions for the problem. Figure 3 displays the geometry and the applied boundary conditions.

![Boundary Conditions](image)

Figure 3 – boundary conditions

In the image, you can observe the defined boundary conditions within the geometry. The left face, highlighted in blue, is fixed, while on the opposite face, we applied a bending moment of 4000 N.mm. As for the mesh structure, we chose to use tetrahedral elements and employed the curvature method to achieve appropriate refinement in the fillet region. Furthermore, the element size on the fillet surface is limited to $r/2$ for the circular fillet and $a/2$ for the elliptical fillet.
4 RESULTS AND DISCUSSION

4.1 MODEL VALIDATION

In order to validate the numerical model, a mesh convergence study was conducted, and a comparison was made with experimental data available in the literature. Figure 4 presents the stress concentration values obtained numerically, in contrast to those obtained by Fessler et al. (1969).

Figure 4 - Stress concentration factor for circular fillet

The average relative error of the numerical factor compared to the literature values was 1.52%, indicating that the modeling parameters and mesh are suitable for describing the physical problem. Equation (1) represents the equation fitted to the values obtained by Fessler et al. (1969), where \( r/d \) belongs to the range 0.005 – 0.040.

\[
K_t = 1,4260 + 0,1643 \cdot \left( \frac{0,1000}{r/d} \right) - 0,0019 \cdot \left( \frac{0,1000}{r/d} \right)^2 \quad (1)
\]
4.2 ANALYSIS OF FILLETS AND STRESS CONCENTRATION

Figure 5 presents the results obtained for the elliptical fillet, using the circular fillet with \( r = 0.4 \) mm from the previously validated model as a basis for comparison. For the purpose of validating the elliptical fillet model, it is noted that at the point where \( a = b = r = 0.4 \) mm, there is a relative error of 4.62% between the results obtained for the elliptical and circular fillets, as indicated by the blue arrow. For numerical simulation purposes, given computational limitations, this dispersion in values is considered acceptable.

The results indicate that, for \( a/b \) ratios greater than 1, stress concentration decreases. Pedersen (2010) also described this behavior for stress concentration under torsional loading, emphasizing the importance of this geometry in static load applications. However, when reducing the ratio, it is observed that stress concentration increases abruptly, suggesting that this geometry intensifies the discontinuity, a phenomenon that was already expected.
In most cases, the maximum stress occurs in the elliptical fillet geometry, except when the $a/b$ ratio is 2. In cases where the maximum stress occurs in the fillet, it can be effectively stated that the use of this structure can replace the traditional circular fillet, provided that the appropriate parameters are applied, which can significantly reduce the maximum stress on the shaft. Figure 6 below shows the distribution of equivalent stress in the geometry with a ratio equal to 2 in the fillet.

![Figure 6 - Stress Distribution in the Keyway with Elliptical Fillet and a/b=2](image)

Source: Author (2023)

It is observed that the maximum stresses are concentrated on the walls of the keyway, reaching slightly higher values than 35 MPa. However, even with the difference in the region where the maximum stress occurs compared to other cases, it is important to note that the elliptical fillet reduces discontinuities and, consequently, stress concentrations.
5 CONCLUSION

This study underscores the power of numerical tools in modeling stress concentration factors in a keyway under bending. In other words, it is possible to obtain values very close to those of other studies for the circular fillet with a reduced cost-to-accuracy ratio, validating the performed simulation. Furthermore, we can conclude that:

● The numerical model successfully achieved the proposed goals of the study.
● Small changes in the fillet’s geometry can reduce stress concentration, as the elliptical fillet with an a/b ratio of 1.25 achieved a 9.98% reduction in the stress concentration factor compared to the circular fillet, resulting in a significant reduction.
● As the a/b ratio increases, the reduction in concentration follows the same trend, with the maximum reduction reaching 24.90%.
● The elliptical fillet model, in all cases, manages to decrease the maximum stresses on the shaft. However, a new study with dynamic loads is necessary to obtain more relevant results for shaft applications.
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