Parametric sensitivity analysis of a one-dimensional thermal-electrical model of a photovoltaic solar module

Análise de sensibilidade paramétrica de um modelo térmico-elétrico unidimensional de um módulo solar fotovoltaico

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ABSTRACT
The efficiency of a photovoltaic module decreases with the increase in the temperature of the photovoltaic cell, which makes the determination of this temperature one of the determining factors in the modeling of a photovoltaic module. The objective of this work is to make a detailed study about the input parameters of a developed thermal-electrical model of a photovoltaic module, based on a sensitivity analysis. The model includes a thermal part, in which the temperature varies along the five layers of the module, and an electrical part, which is based on a single diode five parameters model. The coupling of these models into a thermal-electrical model is described. Through differential sensitivity analysis, the parameters that have a significant impact on estimating the photovoltaic cell temperature were determined. The variations of the sensitivity coefficients throughout a simulated day are presented. Parameters with the highest absolute sensitivity coefficients, such as the transmittance-absorptance product and open circuit voltage, are crucial for accurate temperature estimation and were further researched and determined with greater precision, in order to improve the model reliability. Conversely, parameters with lower
absolute sensitivity coefficients, such as albedo and parallel resistance, were considered less important and could be simplified to enhance computational efficiency.

Keywords: thermal model, electrical model, sensitivity analysis, photovoltaic cell temperature, photovoltaic module.

RESUMO
A eficiência de um módulo fotovoltaico diminui com o aumento da temperatura da célula fotovoltaica, o que torna a determinação desta temperatura um dos fatores determinantes na modelagem de um módulo fotovoltaico. O objetivo deste trabalho é realizar um estudo detalhado sobre os parâmetros de entrada de um modelo térmico- elétrico desenvolvido de um módulo fotovoltaico, baseado em uma análise de sensibilidade. O modelo inclui uma parte térmica, na qual a temperatura varia ao longo das cinco camadas do módulo, e uma parte elétrica, que se baseia em um modelo de diodo único com cinco parâmetros. O acoplamento destes modelos em um modelo térmico-elétrico é descrito. Através da análise de sensibilidade diferencial foram determinados os parâmetros que têm impacto significativo na estimação da temperatura da célula fotovoltaica. As variações dos coeficientes de sensibilidade ao longo de um dia simulado são apresentadas. Parâmetros com os maiores coeficientes de sensibilidade ao longo de um dia simulado são apresentados. Parâmetros com os maiores coeficientes de sensibilidade absoluta, como o produto transmissividade-absorvância e tensão de circuito aberto, são cruciais para uma estimação precisa da temperatura e foram pesquisados e determinados com maior precisão, a fim de melhorar a confiabilidade do modelo. Por outro lado, parâmetros com coeficientes de sensibilidade absoluta mais baixos, como albedo e resistência paralela, foram considerados menos importantes e poderiam ser simplificados para aumentar a eficiência computacional.

Palavras-chave: modelo térmico, modelo elétrico, análise de sensibilidade, temperatura de célula fotovoltaica, módulo fotovoltaico.

1 INTRODUCTION

According to IEA (2023) electricity generation from photovoltaic (PV) solar systems grew 26% in 2022, surpassing wind for the first time in history. In 2027, IEA (2023) predicts that it will outgrow coal. This growing is well on track to reach the Sustainable Development Scenario level by 2030, which will require electricity generation from PV solar modules to increase 15% annually. In order to continue to track this prediction, it is necessary that the research and development of PV modules also grow substantially.

Experimental studies on PV systems are financially expensive and sometimes unfeasible. Therefore, within a first step, it is recommended that studies be directed from
mathematical or computational models, which are able to accurately simulate the behavior of these systems (JAKHRANI, 2013).

Models of PV solar modules can be classified according to the nature of their input parameters, being the most commons the thermal models and the electrical models. Purely electrical models differ from thermal models as they take into account the electrical characteristics of PV modules. Electrical models usually require only two non-electrical inputs: solar irradiance ($G_i$) and PV cell temperature ($T_{cel}$). Unlike thermal models, electrical models do not consider the thermophysical properties of the materials composing the module and therefore don’t make a good estimate of $T_{cel}$.

On the other hand, purely thermal models do not consider the electrical parameters and do not describe correctly the electrical efficiency ($\eta$) variation of the PV module regarding the variation of $G_i$ and $T_{cel}$. Therefore, to correctly describe the behavior of a PV solar module, it is necessary to model its thermal parameters as well as the electrical ones. Within this context of inputs of different natures, a more detailed study such as a parametric study is indicated.

Sensitivity analysis (SA) is a parametric study method that allows you to classify which inputs cause the most sensitivity in the output. The inputs that have the highest dimensionless sensitivity coefficients are those that need to be thoroughly studied, while the parameters that have the lowest dimensionless sensitivity coefficients can be studied with less detail.

The objective of this work is to improve a thermal-electrical model of a photovoltaic solar module, through a parametric study based on a sensitivity analysis. The SA was carried out regarding the outputs $T_{cel}$ and $\eta$, in order to determine which inputs are the most important to the calculation of these outputs.

The rest of this article is structured as follows: Section 2 describe the mathematical modeling of the climatological (2.1), thermal (2.2) and electrical (2.3) models, and their coupling into a thermal-electrical model, in Section 2.4. The sensitivity analysis method is described in Section 2.5. In section 3.1, the inputs that cause more sensitivity in the outputs $T_{cel}$ and $\eta$ are presented, and in Section 3.2 the general results of the thermal-
electric model are presented. In Section 4 the conclusions about the thermal-electrical model and the SA are presented.

2 MATHEMATICAL MODELLING

The thermal-electrical modeling of the PV solar module is divided into two parts, the thermal and the electrical modeling. This section describes each of these modeling and their coupling into a thermal-electrical model. Before, it is described how the irradiance is calculated using average radiation data obtained from climatological websites such as NASA (2023).

2.1 CLIMATOLOGICAL MODEL

The climatological data used in the thermal-electrical model include the monthly average total solar irradiation on a horizontal surface \( \bar{H} \) and the monthly average daily diffuse solar irradiation on horizontal surface \( \bar{H}_d \) in kWh/m²/day, the albedo \( \rho_g \), the mean wind velocity \( V \) in m/s and the mean air temperature \( T_\infty \) in °C. The selected place for the simulation is the Center of Renewable and Alternative Energy (CEAR) in Federal University of Paraíba (UFPB) in João Pessoa, Brazil, of latitude \( \phi = -7.14^\circ \) and longitude \( l = -34.85^\circ \). The PV module in question was considered to be at an angle of \( \beta_i = 10^\circ \) in relation to the horizontal plane, and its collecting face oriented to the geographic north.

The irradiations \( \bar{H} \) and \( \bar{H}_d \) are converted into irradiances via an adaptation of the methodology described in Duffie and Beckmann (2020). Eq. 1 provides the total solar irradiance (in W/m²) onto the tilted surface of the photovoltaic module.

\[
G_i = R_bG_b + G_d \left( 1 + \frac{\cos\beta_i}{2} \right) + G\rho_g \left( 1 - \frac{\cos\beta_i}{2} \right)
\]  

(1)

Where
$R_b$ is the beam solar irradiance elevation factor (dimensionless), $G_b$ is the beam solar irradiance on the horizontal plane (W/m²), $G_d$ is the diffuse solar irradiance on the horizontal plane (W/m²) and $G$ is the total solar irradiance on the horizontal plane (W/m²).

The equation to calculate $R_b$ for a PV module located in the southern hemisphere with the upper face facing north was also taken from Duffie and Beckman (2020):

$$R_b = \frac{\cos(\phi + \beta_i) \cos(\delta) \cos(\omega) + \sin(\phi + \beta_i) \sin(\delta)}{\cos(\phi) \cos(\delta) \cos(\omega) + \sin(\phi) \sin(\delta)}$$  \hspace{1cm} (2)

Where

- $\omega$ is the hour angle of the sun (degrees) and $\delta$ is the solar declination (degrees).

2.2 THERMAL MODEL

For the thermal modeling, a steady state and one-dimensional heat transfer was adopted in the transverse direction to the surface area of the PV module. According to Brano et al. (2014), the simplification of the thermal problem of the module based on the hypothesis of a unidimensionality of the heat flux is justifiable, since the ratio between the total thickness of a module and its surface area tends to be small.

In Figure 1, the temperature of interest to the thermal-electrical model is the temperature of the PV cell $T_{cel}$. To calculate it, an energy balance was carried out that takes into account the energy gains from the irradiance incident on the module glass $G_i$, calculated with Eq.(1), and the portion of solar irradiance that is absorbed by the PV cells $S$, calculated with Eq.(5). The convective energy losses to the ambient air above and below the module and the radiative losses from the upper part of the module to the sky by radiation were also considered in the balance. The radiative losses referring to the lower part of the module to the neighborhood were neglected. The irradiance that is converted into electrical power is represented in Figure 1 by $P_{ele}$, in W/m². Heat losses are represented by the resistances to convection $R^"conv.sup"$ and $R^"conv.inf"$, and the resistance to radiation $R^"rad"$, in m².K/W, in Figure1. The subscripts $sup$ and $inf$ refer to the
upper and lower surfaces of the \( PV \) module. The resistances \( R_{\text{mod, sup}}^\prime \) and \( R_{\text{mod, inf}}^\prime \), also in \( \text{m}^2.\text{K/W} \), represent the equivalent resistances to the heat conduction between the different layers of the module, specified to the left of the thermal circuit in Figure 1. The temperature of the upper surface of the glass is \( T_g \).

From Figure 1, a thermal balance was carried out on the two unknowns, \( T_{\text{cel}} \) and \( T_g \).

Thermal balance on \( T_g \):

\[
\frac{T_\infty - T_g}{R_{\text{conv, sup}}^\prime} + \frac{T_{\text{sky}} - T_g}{R_{\text{rad}}^\prime} + \alpha_g G_i + \frac{T_{\text{cel}} - T_g}{R_{\text{mod, sup}}^\prime} = 0
\]  

\( T_\infty (\degree\text{C}) \) is the temperature of the air and \( \alpha_g \) is the glass solar absorptance. For the sky temperature \( (T_{\text{sky}}, \text{ in } \degree\text{C}) \), it was used the approximation described in Duffie e Beckmann (2020), where \( T_{\text{sky}} = T_\infty - 5 \).
Thermal balance on $T_{cel}$:

$$\frac{T_g - T_{cel}}{R_{mod, sup}^n} + \frac{T_\infty - T_{cel}}{R_{conv, inf}^n + R_{mod, inf}^n} + S = \eta G_i$$  \hspace{1cm} (3)

In Eq.(4), \(\eta\) is the electrical efficiency, calculated with the electrical model. \(S\) (W/m$^2$) is the irradiance that is absorbed by the PV cell. \(S\) is calculated with the Eq.(5),

$$S = (\alpha_{cel} \tau_g)_n \left( R_b K_b G_b + G_d K_d \left( 1 + \frac{\cos \beta_i}{2} \right) + G_p \rho_g K_g \left( 1 - \frac{\cos \beta_i}{2} \right) \right)$$  \hspace{1cm} (4)

Where \((\alpha_{cel} \tau_g)_n\) is the product of the transmittance of the glass with the absorptance of the PV cell for the normal incident solar irradiance, obtained in Notton et al. (2005). The parameters \(K_b, K_d\) e \(K_g\) in Eq.(5), are the incident angle modifiers of the beam, diffuse and ground-reflected solar irradiance, respectively.

2.3 ELECTRICAL MODEL

The electrical model is developed from the electrical circuit that represents a PV module in Figure 2. The PV module circuit includes a current source that represents the photogenerated current ($I_{pv}$), a diode that consumes an amount of current ($I_d$) which includes the reverse saturation current ($I_0$), and two resistances that represent the losses of the PV module. The parallel resistance ($R_p$) represents the losses due to leakage current, which depends on the manufacturing method used, and the series resistance ($R_s$) represents the structural losses of the PV solar module, such as the losses from cable connections and electrical welding (VILLALVA et al., 2009).
Figure 2. Single-diode and two resistances equivalent electrical circuit of a PV module.

\[ I = I_{pv} - I_0 \left[ \exp \left( \frac{V_e + R_s I}{V_t a} \right) - 1 \right] - \frac{V_e + R_s I}{R_p} \]  

(5)

Eq.(6) represents the circuit in Figure 2.

The electrical model developed is based on the model by Villalva et al. (2009). To find out the voltage \( V_e \) and current \( I \) generated by the PV module, it is necessary to first calculate the five unknown parameters in Eq.(6): \( I_{pv}, R_s, R_p, a \) and \( I_0 \). The three parameters reverse saturation current (\( I_0 \)), photogenerated current (\( I_{pv} \)) and diode ideality constant (\( a \)) are functions of the other two unknown parameters, \( R_p \) and \( R_s \). The parameters \( R_p \) and \( R_s \) need an iterative process to be calculated, in which \( R_p \) is calculated for the \( R_s \) values \( 0 < R_s < R_{s,\text{max}} \), until a pair of \( R_p \) and \( R_s \) that makes \( P_{\text{calc}} = P_{\text{stc}} \) is found, where \( P_{\text{stc}} \) is the standard power supplied by the manufacturer and \( P_{\text{calc}} = V_e I \). Finally, the efficiency \( \eta \) is calculated with Eq.(7).

\[ \eta = \frac{P_{\text{max}}}{G_i \cdot \text{area}} \]  

(6)

After calculating \( R_p \) and \( R_s \) for the standard test conditions STC (\( G_i = 1000 \text{ W/m}^2, T_{\text{cel}} = 25 \, ^\circ\text{C} \)), it is possible to calculate \( \eta \) for other climatic conditions beside the STC conditions using the same \( R_p \) and \( R_s \). The parameter \( V_e \) in Eq.(6) could be any value between \( 0 < V_e < V_{e,\text{oc}} \), so it is needed a control method to find the pair of \( V_e \) and \( I \) that calculates the highest power (\( P_{\text{calc}} \)). An easy way to find the highest \( P_{\text{calc}} \) in a
computational model is to calculate all \( P_{\text{calc}} \) for the \( 0 < V_e < V_{e,ac} \) values, and to search for the highest \( P_{\text{calc}} \).

### 2.4 COUPLING OF THE THERMAL AND ELECTRICAL MODELS

![Figure 3. Algorithm of the coupling of the thermal and electrical models.](image)

The first step to calculate the coupled solution of the thermal-electrical model consists of solving the electrical part of the model, in order to calculate an initial value for \( \eta \). For this first solution of \( \eta \), an initial temperature is assigned to the PV cell, \( T_{\text{cel}, \text{initial}} \). Then, the thermal part of the thermal-electrical model is solved, which calculates a \( T_{\text{cel}} \) based on the \( \eta \) previously calculated. Then, the two temperatures, \( T_{\text{cel}} \) and \( T_{\text{cel}, \text{initial}} \), are compared. If the magnitude of the absolute difference between \( T_{\text{cel}} \) and \( T_{\text{cel}, \text{initial}} \) is greater than or equal to the error, \( T_{\text{cel}, \text{initial}} \) becomes equal to \( T_{\text{cel}} \). The electrical model is then restarted. If the new difference is less than the error, the program ends with the final value for \( T_{\text{cel}} \). The basic scheme of the described procedure is shown in Figure 3. The program runs for each of the 78 intervals of 10 minutes between sunrise and sunset, for each simulated day.

### 2.5 SENSITIVITY ANALYSIS

For the sensitivity analysis (SA) made of the developed thermal-electrical model, the average day of November (14/11) was selected. November is the month that presents the highest value of monthly average daily total radiation \( \bar{H} \) for the selected location (NASA, 2021).
The operating scheme of the program developed in MATLAB to perform the SA is shown in Figure 4. First, the thermal-electrical model program runs with the standard inputs, in order to calculate the standard outputs \( \eta_0 \) and \( T_{cel,0} \). The selected input from which the sensitivity analysis will be performed becomes \( x_0 \). \( x_0 \) is incremented for more \( (x_p) \) and for less \( (x_n) \). The thermal-electrical program is executed twice more, once with the input \( x_p \) in order to calculate the outputs \( \eta_p \) and \( T_{cel,p} \), and once again with the input \( x_n \) in order to calculate the outputs \( \eta_n \) and \( T_{cel,n} \). The partial derivatives \( \eta_p \) and \( \frac{\partial \eta}{\partial x} \) and \( \frac{\partial T_{cel}}{\partial x} \) are calculated using the centered finite difference method (ÖZIŞIK et al., 2017).

Afterwards, the partial derivatives are multiplied by the input at his standard value \( (x_0) \), and divided by the standard output value \( (\eta_0 \) or \( T_{cel,0} \)), thus arriving at the sensitivity coefficient of \( \eta \) in relation to the input \( x \) \( (J_{\eta,x}) \), and the sensitivity coefficient of \( T_{cel} \) in relation to \( x \) \( (J_{T_{cel},x}) \). The program comes to an end, and another input parameter can be selected to restart the program. This process is also showed in Figure 4.

The most important parameters of the thermal-electrical model to the \( T_{cel} \) and \( \eta \) outputs, used in the thermal modeling described in Section 0 and in the electrical modeling described in Section 0, are shown in Table 1.
Table 1. Most important inputs of the thermal-electrical model.

<table>
<thead>
<tr>
<th>INITIALS</th>
<th>DESCRIPTION</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_g$</td>
<td>Glass absorptance</td>
<td>0.05 ± 0.0025</td>
</tr>
<tr>
<td>$\varepsilon_g$</td>
<td>Glass emissivity</td>
<td>0.91 ± 0.0455</td>
</tr>
<tr>
<td>$\tau_g\alpha_{cel}$</td>
<td>Transmittance-absorptance product</td>
<td>0.855 ± 0.04275</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>Tilt</td>
<td>10 ± 0.1 °</td>
</tr>
<tr>
<td>$L$</td>
<td>PV module length</td>
<td>1.956 ± 0.0005 m</td>
</tr>
<tr>
<td>$area$</td>
<td>PV module area</td>
<td>1.9404 ± 0.011 m²</td>
</tr>
<tr>
<td>$\bar{H}$</td>
<td>Monthly average daily total radiation</td>
<td>21.924 ± 2.1924 MJ/m².dia (^{(1)})</td>
</tr>
<tr>
<td>$T_{\infty}$</td>
<td>Air temperature</td>
<td>26.61 ± 0.2 °C (^{(1)})</td>
</tr>
<tr>
<td>$V$</td>
<td>Wind velocity</td>
<td>4.45 ± 0.0445 m/s (^{(1)})</td>
</tr>
<tr>
<td>$a$</td>
<td>Diode ideality constant</td>
<td>1.07 ± 0.0535</td>
</tr>
<tr>
<td>$I_{scn}$</td>
<td>Standard short circuit current</td>
<td>9.22 ± 0.3384 A</td>
</tr>
<tr>
<td>$V_{ocn}$</td>
<td>Standard open circuit tension</td>
<td>47 ± 0.5452 V</td>
</tr>
<tr>
<td>$K$</td>
<td>Air thermal conductivity</td>
<td>26.7 ± 1.335 mW/m.K (^{(2)})</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
<td>0.706 ± 0.0353 (^{(2)})</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Air kinematic viscosity</td>
<td>16.5 ± 0.825 .10^{-6} m²/s (^{(2)})</td>
</tr>
</tbody>
</table>

\(^{(1)}\) varies according to the day. The presented value is for 14/11.

\(^{(2)}\) varies along the day. The presented value is for the average of 14/11. Source: Author.

3 RESULTS AND DISCUSSION

The results of the sensitivity analysis (SA) applied on the thermal-electrical model are described in this section. In section 3.1, the inputs that cause more sensitivity in the outputs $T_{cel}$ and $\eta$ are presented. A brief discussion is made about how each of the most important inputs exerts its influence on the outputs. In Section 3.2, the general results of the thermal-electric model applied for the simulation of two days are presented. The results were compared with those of five models found in the literature, three of which are purely thermal models, and two are thermal-electrical models.

3.1 DIFFERENTIAL SENSITIVITY ANALYSIS

The SA program (Figure 4) was executed for each of the 78 intervals of the day for all inputs. Thus, the variations throughout the day of the dimensionless sensitivity coefficients of the $y$ outputs in relation to the $x$ inputs ($J_{y,x}$) were calculated.

Table 2(a) presents the data regarding the most important dimensionless sensitivity coefficients of the $PV$ cell temperature in relation to the $x$ inputs ($J_{T_{cel},x}$). The third column of the Table presents $MEAN(J_{T_{cel},x})$, which represents the average of the...
absolute values of each $J_{T_{cel}x}$ throughout the day. The coefficients $J_{T_{cel}x}$ were sorted in descending order of $MEAN(|J_{T_{cel}x}|)$. Table 2(b) presents the same type of data, but referred to the sensitivity of $\eta$, $J_{T_{cel}x}$ and $MEAN(|J_{T_{cel}x}|)$.

Table 2. Ranking of the most important inputs to the outputs a) $T_{cel}$ and b) $\eta$.

| RANKING | $x$ | $MEAN(|J_{T_{cel}x}|)$ |
|---------|-----|----------------------|
| 1º      | $T_o$       | 0.85226              |
| 2º      | $\tau_{cel}$ | 0.05363              |
| 3º      | $H$         | 0.04228              |
| 4º      | $K$         | 0.02421              |
| 5º      | $e_x$       | 0.01600              |
| 6º      | $V$         | 0.01596              |
| 7º      | $L$         | 0.01593              |
| 8º      | $\nu$       | 0.01210              |
| 9º      | $V_{ocn}$   | 0.00831              |

| RANKING | $x$ | $MEAN(|J_{T_{cel}x}|)$ |
|---------|-----|----------------------|
| 1º      | $V_{ocn}$ | 1.20930              |
| 2º      | $T_c$      | 1.12450              |
| 3º      | $H$        | 0.97311              |
| 4º      | $area$     | 0.86720              |
| 5º      | $I_{scn}$  | 0.81511              |
| 6º      | $a$        | 0.23336              |
| 7º      | $\beta$    | 0.08653              |
| 8º      | $\tau_{g\alpha_{cel}}$ | 0.07534          |
| 9º      | $\beta$    | 0.07301              |

Source: Author

Figure 5 shows the variation of the dimensionless sensitivity coefficients $J_{T_{cel}x}$ throughout the simulated day. The farther the $J_{T_{cel}x}$ is from the axis where $J_{T_{cel}x} = 0$, the more sensitive is the output $T_{cel}$ to the $x$ input. The order of presentation of the $x$ input parameters in the legend of Figure 5(a) and Figure 5(b) is the same order as the ranking of the Table 2(a). After Figure 5, a discussion about each $J_{T_{cel}x}$ is presented.

Figure 5. Sensitivity coefficients of $T_{cel}$ ($J_{T_{cel}x}$) to the variations of the $x$ inputs.

Source: Author
It can be observed in Figure 5 that the dimensionless sensitivity coefficients $J_{T_{cel},x}$ are always more distant from the axis where $J_{T_{cel},x} = 0$ at solar noon. That is, the $J_{T_{cel},x}$ are more significant at this time, with the only exception being the dimensionless sensitivity coefficient of $T_{cel}$ to the input air temperature ($J_{T_{cel},T_{infty}}$), which is more significant at the beginning and end of the day. The increase in the significance of the others $J_{T_{cel},x}$ when close to solar noon occurs because at this time the absolute value of the derivatives $\partial \eta / \partial x$ are greater. And the derivatives $\partial \eta / \partial x$ are greater when close to solar noon due to the increase of $G_i$ and $S$ at this time, in Eq.(1) and Eq.(5).

$T_{infty}$, in Figure 5(a), is the input that most influences the determination of $T_{cel}$. A positive increment in $T_{infty}$ causes a positive increment in $T_{cel}$. In practice, this is because the heat transfer by convection is proportional to the temperature difference between the surfaces of the PV module and $T_{infty}$. In the developed model, $T_{infty}$ is also used to calculate the sky temperature ($T_{sky}$), used in Eq.(3), and the temperature at which the air parameters conductivity ($K$), Prandtl number ($Pr$) and kinematic viscosity ($\nu$) are calculated. $K$, $Pr$ and $\nu$ are used to calculate the convective resistances $R''_{conv, sup}$ and $R''_{conv, inf}$, in Eq.(3) and Eq.(4).

The second highest sensitivity coefficient of $T_{cel}$ is the sensitivity to the variable $\tau_g \alpha_{cel}$, in Figure 5(a). The variable $\tau_g \alpha_{cel}$ represents the fraction of the solar irradiance $G_i$ that is transmitted by the glass and absorbed by the PV cell. An increase in $\tau_g \alpha_{cel}$ causes an increase in the portion of irradiance that is absorbed by the cell ($S$), in Eq.(5), causing a greater portion of the irradiance to be converted into heat in the PV cell, increasing $T_{cel}$. A positive increment in $\tau_g \alpha_{cel}$ causes a positive increment in $T_{cel}$.

The sensitivity coefficient $J_{T_{cel},\bar{H}}$ in Figure 5(a) tells us how sensitive $T_{cel}$ is to the variation of the total horizontal solar irradiation $\bar{H}$. The parameter $\bar{H}$ is used in an adaptation of the methodology described in Duffie and Beckmann (2020), so that the incident irradiance, $G_i$, and the irradiance absorbed by the PV cell, $S$, can be calculated with Eq.(1) and Eq. (5), respectively. A positive increment in $\bar{H}$ causes a positive increment in $T_{cel}$. 
The sensitivity coefficient $J_{T,cel,K}$ in Figure 5(a) tells us how sensitive $T_{cel}$ is to the variation of the thermal conductivity of the air $K$. The parameter $K$ is used to calculate the convective resistances $R_{conv,sup}''$ and $R_{conv,inf}''$, used in the thermal balance of Eq.(3) and Eq.(4). The larger the $K$, the more convective transfer the PV module performs with the ambient air. Therefore, a positive increment in $K$ causes a negative increment in $T_{cel}$, making $J_{T,cel,K}$ negative.

The sensitivity coefficient $J_{T,cel,\varepsilon_g}$ in Figure 5(b) tells us how sensitive $T_{cel}$ is to the variation of the emissivity of the glass $\varepsilon_g$. The parameter $\varepsilon_g$ is used to calculate the radiation resistance in Eq.(3). The larger the $\varepsilon_g$, the more energy the glass loses to the sky by long wavelength radiation, causing the $PV$ module temperatures, which include $T_{cel}$, to decrease. Therefore, a positive increment in $\varepsilon_g$ causes a negative increment in $T_{cel}$, making $J_{T,cel,\varepsilon_g}$ negative.

The sensitivity coefficient $J_{T,cel,V}$ in Figure 5(b) tells us how sensitive $T_{cel}$ is to the variation of the wind speed $V$. The parameter $V$ is used to calculate the convective resistances used in the thermal balances in the Eq.(3) and Eq.(4). The larger the $V$, the more energy the PV module loses by convection. A positive increment in $V$ causes a negative increment in $T_{cel}$, making $J_{T,cel,V}$ negative.

The sensitivity coefficient $J_{T,cel,L}$ in Figure 5(b) tells us how sensitive $T_{cel}$ is to the variation of the length of the $PV$ module $L$. The parameter $L$ is used to calculate the convective resistances used in the thermal balances in the Eq.(3) and Eq.(4). The larger the $L$, the less energy the PV module loses by convection. A positive increment in $L$ causes a positive increment in $T_{cel}$.

The sensitivity coefficient $J_{T,cel,\nu}$ in Figure 5(b) tells us how sensitive $T_{cel}$ is to the variation of the air kinematic viscosity $\nu$. The parameter $\nu$ is used to calculate the convection resistances used in the thermal balances in the Eq.(3) and Eq.(4). The greater the $\nu$, the less energy the $PV$ module loses by convection. A positive increment in $\nu$ causes a positive increment in $T_{cel}$.

The sensitivity coefficient $J_{T,cel,V_{ocn}}$ in Figure 5(b) tells us how sensitive $T_{cel}$ is to the variation of the standard open circuit voltage $V_{ocn}$. In practice, an increase in $V_{ocn}$
causes an increase in the maximum power voltage $V_{mp}$, which causes an increase in the electrical efficiency $\eta$. The increase in $\eta$ makes less absorbed irradiance to be converted into thermal energy, causing $T_{cel}$ to decrease. In the modeling, $V_{ocn}$ is used to calculate the leakage current $I_0$, used in the Eq.(6). The larger the $V_{ocn}$, the smaller is $I_0$, and less $G_i$ is converted into thermal energy, causing $T_{cel}$ to decrease. A positive increment in $V_{ocn}$ causes a negative increment in $T_{cel}$, making $J_{T_{cel}V_{ocn}}$ negative.

The variations of the most important sensitivity coefficients of the output $\eta$ ($J_{\eta,x}$) are shown in Figure 6.

![Figure 6. Sensitivity coefficients of $\eta$ ($J_{\eta,x}$) to the variations of the x inputs.](image)

$V_{ocn}$ in Figure 6(a) is the input that most influences the determination of $\eta$. In practice, an increase in $V_{ocn}$ causes an increase in the maximum power voltage $V_{mp}$, which causes an increase in $\eta$. In the modeling, $V_{ocn}$ is used to calculate the leakage current $I_0$, used in Eq.(6). The larger the $V_{ocn}$, the smaller is $I_0$, causing an increase in $\eta$. A positive increment in $V_{ocn}$ causes a positive increment in $\eta$.

The second highest sensitivity coefficient of $\eta$ is the sensitivity to the variable $T_{\infty}$, in Figure 6(a). An increase in $T_{\infty}$ causes an increase in $T_{cel}$, which causes a decrease in $\eta$. A positive increment in $T_{\infty}$ causes a negative increment in $\eta$, making $J_{T_{cel}T_{\infty}}$ negative.

The sensitivity coefficient $J_{\eta,H}$ in Figure 6(a) tells us how much $\eta$ is sensitive to the variation of the total horizontal solar irradiation $\bar{H}$. $\bar{H}$ is used in an adaptation of the
methodology described in Duffie and Beckmann (2020), so that the incident irradiance $G_i$ can be calculated with Eq.(1). The photogenerated current $I_{pv}$, used in Eq.(6), increases with the growth of $G_i$, causing an increase in $\eta$. A positive increment in $\bar{H}$ causes a positive increment in $\eta$.

The sensitivity coefficient $J_{\eta, area}$ in Figure 6(a) tells us how much $\eta$ is sensitive to the variation of the area of the PV module. The parameter area is used in Eq.(7) to calculate $\eta$. A positive increment in area causes a negative increment in $\eta$, making $J_{\eta, area}$ negative.

The sensitivity coefficient $J_{\eta, I_{scn}}$ in Figure 6(a) tells us how sensitive $\eta$ is to the variation of the standard short circuit current $I_{scn}$. In practice, an increase in $I_{scn}$ causes an increase in the maximum power current $I_{mp}$, which causes an increase in $\eta$. In the modeling, $I_{scn}$ is used to calculate the photogenerated current $I_{pv}$, used in Eq.(6). The larger the $I_{pv}$, the larger is $\eta$. A positive increment in $I_{scn}$ causes a positive increment in $\eta$.

The sensitivity coefficient $J_{\eta, a}$ in Figure 6(b) tells us how much $\eta$ is sensitive to the variation of the diode ideality constant $a$. The larger the parameter $a$, less idealized the diode is, and less current the module generates for a given voltage. A positive increment in $a$ causes a negative increment in $\eta$, making $J_{\eta, a}$ negative.

The sensitivity coefficient $J_{\eta, \beta_i}$ in Figure 6(b) tells us how much $\eta$ is sensitive to the variation of the slope of the PV module. The simulated PV module has a slope of $\beta_i = 10^\circ$ from the horizontal, with the collector facing the geographical north. The latitude of the simulation site is $\phi = -7.14^\circ$, while the solar declination on the day selected for the analysis (11/14) is $\delta = -19^\circ$. That is, while the PV module is tilted to the north, the Sun makes its apparent trajectory to the south of the simulation site. By increasing $\beta_i$, the solar irradiance becomes even less perpendicularly on the PV module, causing a decrease in $G_i$ and $\eta$. A positive increment in $\beta_i$ causes a negative increment in $\eta$, making $J_{\eta, \beta_i}$ negative.

The sensitivity coefficient $J_{\eta, \tau_g \alpha_{cel}}$ in Figure 6(b) tells us how much $\eta$ is sensitive to the variation of the product of the transmittance of the glass with the absorptance of the PV cell, $\tau_g \alpha_{cel}$. The parameter $\tau_g \alpha_{cel}$ is used to calculate the amount of irradiance
that is absorbed by the cells (S) in Eq.(5). The increase in S, which is caused by the increase in \( \tau_g \alpha_{cel} \), causes an increase in \( T_{cel} \), which causes a decrease in \( \eta \). A positive increment in \( \tau_g \alpha_{cel} \) causes a negative increment in \( \eta \), making \( J_{\eta, \tau_g \alpha_{cel}} \) negative.

The sensitivity coefficient \( J_{\eta, \beta} \) in Figure 6(b) tells us how sensitive \( \eta \) is to the variation of the thermal coefficient of the open circuit tension \( \beta \). The parameter \( V_{ocn} \) is used to calculate the generated voltage \( V \). The greater the \( \beta \), the less \( V \) is generated for a given temperature and the lower is the efficiency \( \eta \). A positive increment in \( \beta \) causes a negative increment in \( \eta \), making \( J_{\eta, \beta} \) negative.

3.1.1 Discussion about the Sensitivity Analysis

Sensitivity analysis is a useful tool to help make decisions about a model. The SA results serve as a justification for using or not using a consideration. For example, the albedo (\( \rho \)) could be considered as constant for every day, instead of varying it every day. \( \rho \) is the parameter that causes the least sensitivity in \( T_{cel} \) so it does not appear in the ranking of the most important inputs on Table 2. A 30\% change in \( \rho \) results in a difference of only 0.09\% in the maximum \( T_{cel} \) (\( T_{cel,max} \)) and 0.03\% in the maximum \( \eta \) (\( \eta_{max} \)) on 11/14.

A consideration that is made in simpler electrical models is to consider \( R_p \) to be very large (infinite), and \( R_s \) to be very small (zero). \( R_p \) and \( R_s \) are also not on the ranking of the most important inputs on Table 2. In order to prove this consideration, the model was simulated with an \( R_s \) 100 times smaller than the standard \( R_s \) and an \( R_p \) 100 times bigger than the standard \( R_p \). This simulation presented a \( T_{cel,max} \) 0.19\% higher than the standard \( T_{cel,max} \), and a \( \eta_{max} \) 1.64\% higher than the standard \( \eta_{max} \). In the developed model, the pair of resistances \( R_p \) and \( R_s \) was kept constant for all days. Since \( R_p \) and \( R_s \) are not so important inputs for the calculation of \( T_{cel} \) and \( \eta \), the consideration of keeping \( R_p \) and \( R_s \) constant for all days can be made without causing a considerable numerical difference. But it is suggested that \( R_p \) and \( R_s \) should be calculated and not considered, respectively, as infinite and zero.
The SA was made for one day, 11/14. Due to the non-linearity of the system, the SA results if it were performed on other days would be different. For example, the sensitivity coefficient of \( \eta \) in relation to the slope of the PV module \( J_{\eta,\beta_i} \), shows great variability if calculated for different days. When solar declination \( \delta \) is greater than latitude \( \phi \) \((\delta > \phi)\), \( J_{\eta,\beta_i} \) is positive, and when \( \delta < \phi \), \( J_{\eta,\beta_i} \) is negative. Therefore, care must be taken and the results of the SA should only be used on the day that the SA was performed.

3.2 RESULTS OF THE THERMAL-ELECTRICAL MODEL

The results of the Sensitivity Analysis allowed us to establish which input parameters are the most important to the developed thermal-electrical model, so that the uncertainties of these parameters could be determined with more precision than the uncertainties of the less important parameters. With the most important uncertainties established (Table 1), it is possible to determine the total uncertainty of the model’s outputs \( T_{cel} \) and \( \eta \). Figure 7 and Figure 8 presents the results of the thermal-electrical model for the average days that represent the summer and winter solstices months, 11/06 \((T_{\infty} = 25.61 \, ^{\circ}C, \, V = 4.44 \, m/s, \, G_{i,max} = 640.35 \, W/m^2)\) and 10/12 \((T_{\infty} = 27.07 \, ^{\circ}C, \, V = 4.30 \, m/s, \, G_{i,max} = 796.72 \, W/m^2)\), where \( G_{i,max} \) is the maximum irradiance found at solar noon.

The results of \( T_{cel} \) and \( \eta \) of the developed model, presented as \( TE \, Haas \) in Figure 7 and Figure 8, were plotted as a function of the legal time of day - from sunrise to sunset, in 78 intervals of 10 minutes. The results were compared with the results of five other models found in the literature, three of them being the purely thermal models \( T \, NOCT \), \( T \, Risser \) and \( T \, Ross \) (ROSS (1976), RISSE and FUENTES (1984) and ROSS (1986)), and the two thermal-electrical models \( TE \, King \) and \( TE \, Smets \) (KING et al. (2004) and SMETS et al. (2016)).
Figure 7. Results of the thermal-electrical model (TE Haas) for 11/06, compared with other five models, for a) $T_{\text{cel}}$ and b) $\eta$.

![Figure 7](image_url)

Source: Author

It can be seen in Figure 7(a) and Figure 8(a) that the minimum $T_{\text{cel}}$ ($T_{\text{cel, min}}$) is always found at the beginning and end of the day for all models. The purely thermal models $T_{\text{NOCT}}$ and $T_{\text{Ross}}$ and the thermal-electrical model $TE_{\text{King}}$ have $T_{\text{cel, min}} = T_\infty$, while the $T_{\text{Risser}}$ model has $T_{\text{cel, min}} > T_\infty$. The $TE_{\text{Haas}}$ and $TE_{\text{Smets}}$ models have $T_{\text{cel, min}} < T_\infty$ because at this time they lose thermal energy via radiation to the sky. The $T_{\text{cel}}$ of the $T_{\text{NOCT}}$ and $T_{\text{Ross}}$ models are within the error margins of the $TE_{\text{Haas}}$ model for most of the day, while the $T_{\text{cel}}$ of the $T_{\text{Risser}}$ model is almost half of the day within the $TE_{\text{Haas}}$ margins of error.

Pure thermal models make a good estimate of the $PV$ cell temperature. However, the effect of the $T_{\text{cel}}$ variation alone is not enough to correctly predict the variation in the efficiency throughout the day, in Figure 7(b) and Figure 8(b). The efficiency of the purely thermal models $T_{\text{NOCT}}$, $T_{\text{Risser}}$ and $T_{\text{Ross}}$ only take into account the influence of $T_{\text{cel}}$ in $\eta$, disregarding the influence of $G_i$ in $\eta$. The $T_{\text{cel}}$ of these models present their highest values at the beginning and at the end of the day, when $T_{\text{cel, min}}$ is found. However, at these times, when there is no solar radiation, the current $I_{pv}$ is equal to 0, which makes $\eta$ equal to 0 as well (DURISCH, 2007). Thus, it is expected that models that start the day with $\eta = 0$, like the $TE_{\text{Haas}}$ and $TE_{\text{Smets}}$ models, tend to better describe the behavior of $\eta$ throughout the day. The $TE_{\text{Haas}}$ and $TE_{\text{Smets}}$ models take more into account the
influence of $G_i$ on $\eta$ than the influence of $T_{\text{cel}}$ on $\eta$, causing the largest $\eta$ to be found at solar noon. The efficiency of the $TE$ $King$ model is higher than the efficiency given by the manufacturer $\eta_{\text{stc}} = 17.3\%$, indicating that this model overestimates the efficiency. None of the efficiencies of the five models compared with the developed model are within the error margins of the calculated efficiency $T_{\text{cel}}$.

Figure 8. Results of the thermal-electrical model ($TE$ $Haas$) for 10/12, compared with other five models, for a) $T_{\text{cel}}$ and b) $\eta$.

Source: Author

4 CONCLUSION

Photogenerated energy varies according to the climatic conditions of the installation site, and the study of this variation deserves attention. In this work, a mathematical model of a $PV$ module was developed in order to obtain more realistic data regarding the generated energy. The model is composed of a thermal part, which has thermal parameters as input, and an electrical part which has electrical parameters as input, in order to estimate the temperature of the $PV$ cell $T_{\text{cel}}$ and the efficiency $\eta$. The coupling of the two parts into a thermal-electrical model was described in Section 0. The results for $T_{\text{cel}}$ and $\eta$ were presented and compared with five other models in section 3.2. The fact that many thermal models are within the margins of the $T_{\text{cel}}$ calculated by the developed model in Figure 7(a) and Figure 8(a) indicate that this model makes a good estimate of $T_{\text{cel}}$. And as discussed in section 3.2, the shapes of the efficiency curves in
Figure 7(b) and Figure 8(b) of the developed model indicate that it describes correctly the behavior of $\eta$ throughout the day.

The sensitivity analysis performed in Section 0 helps to define which inputs are most important to the model. They are described in Table 2. These inputs were researched and determined more precisely in comparison to the parameters that the $SA$ defined as being less important. In Section 0, a brief discussion was made about some considerations that were corroborated by the $SA$. The parametric study made generated more knowledge about the functioning of the model, making it more reliable.
REFERENCES


